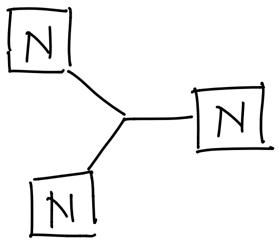
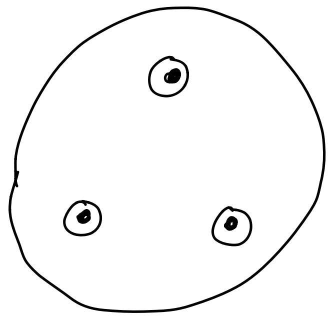
T_N theory:





no marginal deformations

flavor symmetry: $SU(N)^3$

Coulomb branch: dim-k operators $u_k^{(i)}$ for

$k = 3, 4, \ldots, N$

$i = 1, 2, \ldots, k-2$

Higgs branch: dim-2 operators μ_a $a = 1, 2, 3$ in the adjoint of a-th $SU(N)$

dim-$(N-1)$ operators Q_{ijk} and $\tilde{Q}^{ijk}$ in (N, N, N) and $(\bar{N}, \bar{N}, \bar{N})$ of $SU(N)^3$ symmetry.

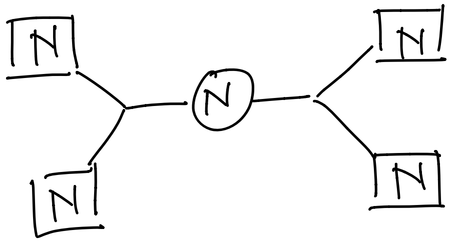
charges of operators:

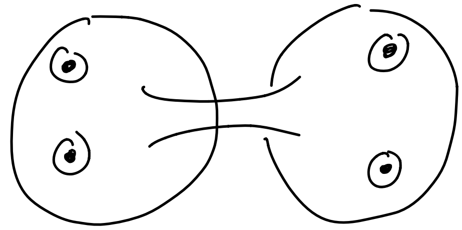
$$R_{\mathcal{N}=1} = \frac{1}{3} R_{\mathcal{N}=2} + \frac{4}{3} I_3 \,, \quad \mathcal{J} = R_{\mathcal{N}=2} - 2 I_3$$

flavor symmetry

$$\mathcal{J}(u_k^{(i)}) = 2k, \quad \mathcal{J}(\mu_i) = -2, \quad \mathcal{J}(Q_{ijk}) = \mathcal{J}(\tilde{Q}^{ijk}) = -(N-1)$$

gauging:





Sicilian gauge theories:

Consider $2n$ copies of T_N theory

→ $SU(N)^{6n}$ flavor symmetry

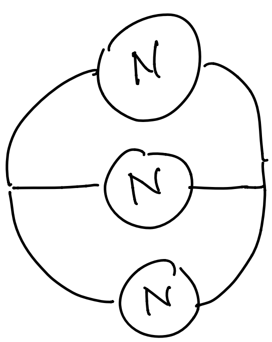
gauging all flavor symmetries gives

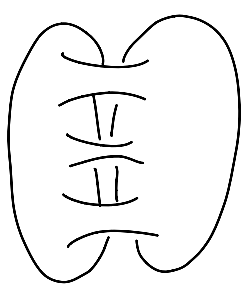
$3n$ gauge nodes

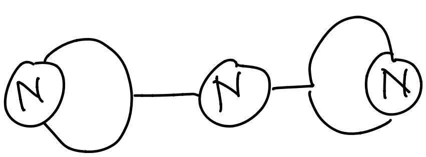
$\parallel$

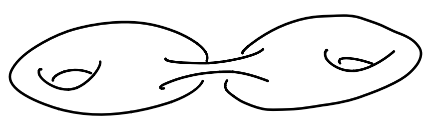
$3g-3$

→ N M5-branes wrapped on genus g "T_g theory"
Riemann surface Σ_g









→ β-function vanishes:

$$\beta = \underbrace{3T(adj) - T(adj)}_{\mathcal{N}=2 \text{ VM}} - \underbrace{2\left[k_{SU(N)}/2\right]}_{\substack{\text{flavor sym.}\\ \text{of } T_N \text{ theory}}} = 0$$

<u>Deformation by adjoint mass → $\mathcal{N}=1$</u>:

superpotential:

$$W = \sum_s \mathrm{tr}\left(\Phi_s \mu_{a(s), i(s)}\right) - \mathrm{tr}\left(\Phi_s \mu_{b(s), j(s)}\right)$$

where Φ_s $(s=1,\dots,3n)$ is the adjoint scalar in the s-th $SU(N)$ VM and $\mu_{a,i}$ $(a=1,\dots,2n,$ $i=1,2,3)$ is the chiral operator in the adjoint of the i-th $SU(N)$ flavor symmetry of the a-th T_N theory

→ add mass term for adjoint chiral Φ_s:

$$W_m = \sum_s m_s^2 \,\mathrm{tr}\, \Phi_s^2$$

→ integrate out Φ_s:

$$\sum_s \frac{1}{m_s} \mathrm{tr}\left(\mu_{a(s), i(s)} - \mu_{b(s), j(s)}\right)^2$$

→ $\mu_{a,i}$ have dim $\frac{3}{2}$ in IR SCFT

Using $R[\mathcal{O}] = \frac{2}{3}\Delta[\mathcal{O}] = \frac{2}{3}\left(\Delta_{UV}[\mathcal{O}] + \frac{\gamma[\mathcal{O}]}{2}\right)$

and $\Delta_{UV}[\mu] = 2$

we get

$$R[\mu] = \frac{2}{3}\left(2 + \frac{\gamma[\mu]}{2}\right) \overset{!}{=} 1$$

$$\Rightarrow \gamma[\mu] = -1$$

Since $\mathcal{J}(\mu_i) = -2 \longrightarrow \gamma = \frac{\mathcal{J}}{2}$

$$\rightarrow \mathrm{tr}\left(\gamma T^a T^b\right) = \frac{1}{2}\mathrm{tr}\left(\mathcal{J} T^a T^b\right) = -\frac{K_{SU(N)}}{4}\delta^{ab}$$

$$\rightarrow \beta = \underbrace{3T(adj)}_{\mathcal{N}=1\ VM} - \underbrace{2\left[K_{SU(N)}/2\right]}_{2\times T_N} - \underbrace{2\left[K_{SU(N)}/4\right]}_{2\times\gamma\{T_N\}} = 0$$

<u>Counting exactly marginal deformations</u>:

$\mathcal{T}_g$ theory after $\mathcal{N}=1$ deformation has $6n-1$ exactly marginal couplings:

- $3n$ from gauge couplings of original $\mathcal{N}=2$ theory
- $3n-1$ from ratios of mas parameters
- one more

$$\longrightarrow \dim_{\mathbb{C}} \mathcal{M}_C = 6n$$

Compactification of M5 branes on Σ_g:

To preserve SUSY, one embeds $SO(2)$ spin connection of Σ_g into $SO(5)$ R-sym of $(2,0)$ theory:

$$SO(2) \times SO(3) \subset SO(5)$$

$\rightarrow$ commutant is $U(1) \times SU(2)$ R-sym of $\mathcal{N}=2$ SCFT in 4d

To get $\mathcal{N}=1$ SCFT, decompose

$$SU(2) \times SU(2)_F \simeq SO(4) \subset SO(5)$$

and embed spin connection in $U(1)_R \subset SU(2)$

$\rightarrow$ 4 of $SO(5)$ decomposes under $U(1)_R \times SU(2)_F$ as $\quad 4 \longrightarrow 1_+ \oplus 1_- \oplus 2_0$

$\rightarrow$ twisting makes 1_+ covariantly constant

$\rightarrow$ $U(1)_R \subset SU(2)$ gives $U(1)$ R-symmetry of the $\mathcal{N}=1$ SCFT

$SU(2)_F$ remains unbroken $\rightarrow$ flavor sym.

marginal deformations:

$$\left.\begin{array}{l} \bullet\ 3g-3 = 3n \text{ moduli from } \Sigma_g \\ \bullet\ SU(2)_F \text{ Wilson lines: } 3g-3 \end{array}\right\} \begin{array}{l} \text{in total} \\ 6g-6 \\ = 6n \end{array}$$